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Temporal evolution of the bright soliton in an optical fiber

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Abstract. – The bright soliton in an optical fiber is generally studied by treating the spatial variable as the evolution variable, where the waveform is taken into consideration. To be consistent with the well-established picture of the dynamics of solitons in other systems, in this letter, we propose that it is helpful to take the temporal variable as the evolution one and study the waveshape of the bright soliton via its propagation along the space coordinate axis. We develop a singular theory. Equations governing the evolution of parameters of the bright soliton in the slow time and the radiated field are explicitly formulated for the first time.

Owing to the promising application to long-distance soliton-based communication and the great fundamental physical interest, solitary waves and solitons in a nonlinear monomode optical fiber have received intensive studies in recent years [1,2]. The generalized propagation equation of the optical field in a fiber takes the form

$$iu'_{x'} + ik_1 u'_{t'} - \frac{1}{2}k_2 u'_{t't'} + \sigma \left|u'\right|^2 u' = i\varepsilon P'\left[u'\right], \tag{1}$$

where x' represents the propagation distance, t' the time and u' the complex field envelope. Usually, $\varepsilon P'[u']$, including linear loss, high-order dispersion and other nonlinear effects, is assumed to be small and treated as perturbations to place emphasis on the study of important phenomena of bright and dark solitons in an optical fiber [3]. In the region of anomalous group-velocity dispersion, by introducing the retarded time $T' = t' - k_1 x' = t' - x'/v_g$, eq. (1) is normalized as

$$iu''_{x''} + \frac{1}{2}u''_{TT} + \left|u''\right|^2 u'' = i\varepsilon P''\left[u''\right] \tag{2}$$

in terms of $T = T'/T_0$, $x'' = x'/L_D = x'\left|k_2\right|/T_0^2$ and $u'' = \sqrt{\left|k_2\right|/\sigma T_0^2}\,u'$ [1]. Equation (2) is referred to as the optical nonlinear Schrödinger equation (NLSE), and its unperturbed version supports steady propagation of a type of solitary wave called the bright soliton [3].

Generally, waves traveling along the x-axis at speed v are expressible as functions of $(x-vt)$. A wave $F(x,t)$ may be thought of as formed from the shape $f(\zeta)$ by the substitution $\zeta = (x-vt)$, or else as built from the time signal $h(\tau)$ by the substitution $\tau = (t-x/v)$. Here, $f(\zeta)$ which satisfies $f(x) = F(x,0)$ characterizes the "waveshape", and $h(\tau)$ which satisfies

$h(t) = F(0,t)$ depicts the "waveform" [4]. Pictures resulted from the two standpoints for the wave $F(x,t)$ are that the waveshape propagates and changes along the x-axis as time elapses and the waveform varies *vs.* the retarded time τ as the distance x keeps increasing. These actually present two different points of view for the visualization of the scenario of a soliton under perturbations.

The bright soliton propagating in an optical fiber governed by eq. (2) was typically investigated by interchanging the roles of the retarded time T and the space x'' and defining an "initial-value" problem, or equivalently by directly treating the space x'' as the evolution variable and defining a boundary value problem. Accordingly, the waveform of a bright soliton was taken into consideration and studies could benefit from the direct application [5, 6] of the celebrated frameworks developed by Zakharov and Shabat (ZS) and by Ablowitz-Kaup-Newell-Segur (AKNS). Heuristically, to avoid complication of the ZS and AKNS schemes, other elaborate approaches were also developed in the framework of direct expansion, presenting excellent theoretical paradigms for the subject [7, 8]. However, in contrast with studies of the waveform of the bright soliton, the waveshape is extensively investigated in other problems of solitons under perturbations [6], including the envelope soliton of the integrable cubic NLSE in water and other applications [9, 10]. Although knowledge for understanding evolution of the waveform of a bright soliton in a fiber has been gained from the above-mentioned theories, a natural question, how the waveshape evolves in the real time, inevitably arises. To answer this question, the corresponding mathematical model is essentially different from the one in the previous studies and is also intractable in the ZS and AKNS schemes. Consequently, a new theoretical challenge turns up. In this letter, we develop our theory for the subject.

Let us start from a dimensionless form of eq. (1) in the anomalous dispersion regime

$$iu_x + iu_t + \frac{1}{2}u_{tt} + |u|^2 u = i\varepsilon P[u], \tag{3}$$

where we put $t = t'/t_0$, $x = x'/l = x'|k_2|/t_0^2$, $u = \sqrt{|k_2|/\sigma t_0^2}u'$ and $t_0 = |k_2|/k_1 = v_g^{-1}|\mathrm{d}v_g/\mathrm{d}\omega|$. Apparently, instead of the usual T_0 determined by the width of the input waveform in the previous theories [1], a characteristic time, namely t_0, that is determined by the working wavelength and the nature of fibers, is introduced in the normalization. Formally, eq. (3) differs from eq. (2), that is the normal form of the optical NLSE, only by an additional term iu_t due to the absence of the retarded time. In fact, the essential difference lies in that the time t here must be treated as an evolution variable and an initial-value problem must be defined since the waveshape of a bright soliton is to be taken into account. Now, we specify the problem we are to tackle. It is known that, in the absence of perturbations, eq. (3) admits a general bright soliton solution characterized by four parameters in the form

$$u_{\mathrm{sol}}(x,t) = 2\eta\,\mathrm{sech}\,2\eta(2\zeta+1)\left[x - \frac{1}{(2\zeta+1)}t - \chi'\right] \times \exp\left[-i[2(\zeta^2-\eta^2+\zeta)x - 2\zeta t - \theta_1]\right], \tag{4}$$

provided that the initial waveshape is given by

$$u_{\mathrm{sol}}(x,0) = 2\eta\,\mathrm{sech}\,2\eta(2\zeta+1)(x-\chi') \times \exp\left[-i[2(\zeta^2-\eta^2+\zeta)x - \theta_1]\right]. \tag{5}$$

However, if the perturbations turn on, the bright soliton beginning with an initial state of eq. (5) and then propagating in an optical fiber governed by eq. (3) cannot be described by eq. (4). Thus, we must derive the solution of eq. (3) under the initial condition of eq. (5). To the best of our knowledge, the problem noted here still remains untouched. Moreover, it turns out to be a challenging one of dynamical significance.

From the general point of view of solitons under perturbations, the system described by eq. (3) is still dominated by a soliton. In addition, other wave modes of smaller amplitude may appear. Moreover, it is important to point out that the soliton will undergo a slow change, depicted by its varying parameters [6,11]. To characterize such a picture, we introduce a slow time scale $t_1 = \varepsilon t$ and assume that the solution of eq. (3) is of the form

$$u(t, z, t_1) = [2\eta h(z) + \varepsilon v(t, z, t_1)]e^{-i\theta(z,t_1)}, \tag{6}$$

where $h(z) = \mathrm{sech} z$, $z = 2\eta(2\zeta + 1)(x - \varepsilon^{-1}\chi - \chi')$ and $\theta = (Kz - \varepsilon^{-1}\theta_0 - \theta_1)$. Further, we assume that η, ζ, χ, χ', K, θ_0, θ_1 are directly dependent on t_1. Obviously, z is the coordinate variable in the reference frame tied up to the bright soliton.

Now, if we take t, z and t_1 as new independent variables in place of t and x, the derivatives with respect to time and space in eq. (3) are thus replaced by

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t} - 2\eta(2\zeta + 1)\Lambda\frac{\partial}{\partial z} + \varepsilon\frac{\eta_{t_1}}{\eta}z\frac{\partial}{\partial z} + \varepsilon\frac{2\zeta_{t_1}}{(2\zeta + 1)}z\frac{\partial}{\partial z} - \varepsilon 2\eta(2\zeta + 1)\chi'_{t_1}\frac{\partial}{\partial z} + \varepsilon\frac{\partial}{\partial t_1} \tag{7}$$

and

$$\frac{\partial}{\partial x} = 2\eta(2\zeta + 1)\frac{\partial}{\partial z}, \tag{8}$$

where $\chi_{t_1} = \Lambda$ is defined. Inserting eqs. (6)-(8) into eq. (3), we transform from the laboratory frame into the soliton's one and get two equations from $O(1)$ and $O(\varepsilon)$, respectively. The zeroth-order equation for $O(1)$ yields $\Lambda = (2\zeta + 1)^{-1}$, $\theta_{0t_1} = \Omega = 2(\zeta^2 + \eta^2)\Lambda$ and $K = (\zeta^2 - \eta^2 + \zeta)\Lambda\eta^{-1}$. With these relations, we simplify the first-order equation for $O(\varepsilon)$ as

$$\frac{1}{2}v_{tt} + i\Lambda^{-1}v_t - 2\eta v_{zt} + 2\eta^2 v_{zz} + 8\eta^2 h^2(z)v + 4\eta^2 h^2(z)v^* - 2\eta^2 v = R(z), \tag{9}$$

where the asterisk "$*$" denotes the complex conjugate and the so-called "source term" [11] is given by $R(z) = R_r(z) + iR_i(z)$, with $R_r(z)$ and $R_i(z)$ determined by

$$\begin{aligned} R_r \;=\; & -\mathrm{Im}(Pe^{i\theta}) - 4\eta(2\eta_{t_1} + \Lambda\eta\zeta_{t_1})\varphi_2(z) + 16\eta^3\Lambda^{-1}\chi'_{t_1}\phi_1^3(z) - 8\eta[2\Lambda\eta\zeta_{t_1} + \eta_{t_1}]z\phi_1^3(z) + \\ & +2\eta\Lambda^{-1}[2\eta(\Lambda^{-1}K - 2\eta)\chi'_{t_1} + \theta_{1t_1}]\phi_1(z) - 2[(\Lambda^{-1} - 4\eta^2\Lambda)\zeta_{t_1} - 4\eta\eta_{t_1}]\phi_2(z) \end{aligned} \tag{10}$$

and

$$\begin{aligned} R_i \;=\; & \mathrm{Re}(Pe^{i\theta}) - 4\eta^2[\theta_{1t_1} + \Lambda^{-1}(2\eta K + \Lambda^{-1})\chi'_{t_1}]\varphi_2(z) + \\ & +4\eta(\zeta_{t_1} - \Lambda\eta\eta_{t_1})\phi_1(z) - 2[(\Lambda^{-1} - 4\eta^2\Lambda)\eta_{t_1} + 4\eta\zeta_{t_1}]\varphi_1(z). \end{aligned} \tag{11}$$

Here, $\phi_1(z) = \mathrm{sech} z$, $\phi_2(z) = z\,\mathrm{sech} z$, $\varphi_1(z) = \mathrm{sech} z(1 - z\tanh z)$ and $\varphi_2(z) = \mathrm{sech} z \tanh z$ are defined for simplicity and later use. As might have been expected, a novel equation emerges after the linearization. We note that, although the basic idea of the present linearization is a natural extension of the normal scheme of multiple scale expansion [12], the implementation in handling soliton bearing equations with a second-order derivative with respect to time is original. It is successful in our recent study on the sine-Gordon model [13]. As usual, extra freedoms for the purpose of preventing the occurrence of secular terms are introduced and included in the source term. Solving eq. (9) by the Laplace transform yields

$$\frac{1}{2}s^2\widetilde{v} + i\Lambda^{-1}s\widetilde{v} - 2\eta s\widetilde{v}_z + 2\eta^2\widetilde{v}_{zz} + 8\eta^2 h^2(z)\widetilde{v} + 4\eta^2 h^2(z)\widetilde{v}^* - 2\eta^2\widetilde{v} = s^{-1}R(z), \tag{12}$$

where $\widetilde{v}$ stands for the Laplace transform of v. Putting $v = v_1 + iv_2$ and $\widetilde{v} = \widetilde{v}_1 + i\widetilde{v}_2 = (w_1 + iw_2)e^{\frac{sz}{2\eta}}$, we derive

$$sw_1 + 2\eta^2\Lambda\widehat{L}_1 w_2 = s^{-1}\Lambda R_i e^{-\frac{sz}{2\eta}}\,, \tag{13}$$

$$sw_2 - 2\eta^2\Lambda\widehat{L}_2 w_1 = -s^{-1}\Lambda R_r e^{-\frac{sz}{2\eta}}\,, \tag{14}$$

from the real and imaginary parts of eq. (12), where two Hermitian operators $\widehat{L}_1 = \mathrm{d}^2/\mathrm{d}z^2 + (2\,\mathrm{sech}^2 z - 1)$ and $\widehat{L}_2 = \mathrm{d}^2/\mathrm{d}z^2 + (6\,\mathrm{sech}^2 z - 1)$ are defined. To solve eqs. (13) and (14) by making use of the method of eigen-expansion, a complete set of basis is needed. Considering the homogeneous counterpart of eqs. (13) and (14), we derive the following eigenvalue problem:

$$\widehat{L}_1\phi = \lambda\varphi\,, \tag{15}$$

$$\widehat{L}_2\varphi = \lambda\phi. \tag{16}$$

Here, if we define a non-Hermitian operator $\widehat{H} = \widehat{L}_2\widehat{L}_1$, then the corresponding adjoint operator reads $\widehat{H}^\dagger = \widehat{L}_1\widehat{L}_2$. Using the operator $\widehat{L}_2$ to act on both sides of eq. (15) and then the $\widehat{L}_1$ on eq. (16) gives

$$\widehat{H}\phi = \lambda^2\phi\,, \tag{17}$$

$$\widehat{H}^\dagger\varphi = \lambda^2\varphi. \tag{18}$$

The eigenstates of operators $\widehat{H}$ and $\widehat{H}^\dagger$ consist of a continuous spectrum with eigenvalue $\lambda = -(k^2+1)$ and doubly degenerated discrete states with eigenvalue $\lambda=0$, respectively. Under the definition of the inner product in the Hilbert space, the eigenstates $\phi=\{\phi(z,k),\phi_1(z),\phi_2(z)\}$ and $\varphi=\{\varphi(z,k),\varphi_1(z),\varphi_2(z)\}$ turn out to be a biorthogonal basis (BB), which satisfies the completeness relation

$$\int_{-\infty}^{+\infty}\phi(z,k)\varphi^*(z',k)\mathrm{d}k + \phi_1(z)\varphi_1(z') + \phi_2(z)\varphi_2(z') = \delta(z-z'), \tag{19}$$

where

$$\phi(z,k) = \frac{1}{\sqrt{2\pi}(k^2+1)}(1 - 2ik\tanh z - k^2)e^{ikz} \tag{20}$$

and

$$\varphi(z,k) = \frac{1}{\sqrt{2\pi}(k^2+1)}(1 - 2\mathrm{sech}^2 z - 2ik\tanh z - k^2)e^{ikz} \tag{21}$$

represent the continuous spectrum and $\phi_1(z)$, $\phi_2(z)$, $\varphi_1(z)$, $\varphi_2(z)$ that are defined above stand for the discrete states. BB is popular in the studies of non-Hermitian Hamiltonian problems [14]. Taking advantage of the set of BB, we expand the solutions of eqs. (13) and (14) as

$$w_1(t,z,t_1) = \int_{-\infty}^{+\infty}\widetilde{w}_1(t,k,t_1)\varphi(z,k)\mathrm{d}k + \widetilde{w}_{11}(t,t_1)\varphi_1(z) + \widetilde{w}_{12}(t,t_1)\varphi_2(z)\,, \tag{22}$$

$$w_2(t,z,t_1) = \int_{-\infty}^{+\infty}\widetilde{w}_2(t,k,t_1)\phi(z,k)\mathrm{d}k + \widetilde{w}_{21}(t,t_1)\phi_1(z) + \widetilde{w}_{22}(t,t_1)\phi_2(z). \tag{23}$$

Introducing eqs. (22) and (23) into eqs. (13) and (14) and solving by means of orthogonality of the basis, we derive w_1 and w_2. Thus, v_1 and v_2 are determined from the inverse Laplace transformation. Some terms directly proportional to t and t^2 are found to appear in v_1 and v_2, they are nonphysical and called secular terms. But if we require

$$\int_{-\infty}^{+\infty} R_i(z)\phi_1(z)\mathrm{d}z = 0, \qquad \int_{-\infty}^{+\infty} R_i(z)\phi_2(z)\mathrm{d}zz + 2\eta\Lambda \int_{-\infty}^{+\infty} R_r(z)z\varphi_2(z)\mathrm{d}z = 0\,, \tag{24}$$

$$\int_{-\infty}^{+\infty} R_r(z)\varphi_2(z)\mathrm{d}z = 0, \qquad \int_{-\infty}^{+\infty} R_r(z)\varphi_1(z)\mathrm{d}z + 2\eta\Lambda \int_{-\infty}^{+\infty} R_i(z)z\phi_1(z)\mathrm{d}z = 0, \tag{25}$$

they vanish. Then we get the final solution

$$\begin{aligned} v_1 \;=\; & \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \frac{1}{2\eta^2\lambda}(\sin\beta)R_i(z')\phi^*(z',k)\varphi(z,k)\mathrm{d}z'\mathrm{d}k + \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \frac{1}{2\eta^2\lambda}(1-\cos\beta)\times \\ & \times R_r(z')\varphi^*(z',k)\varphi(z,k)\mathrm{d}z'\mathrm{d}k - \int_{-\infty}^{+\infty} \frac{\Lambda}{2\eta}R_i(z')z'\phi_1(z')\mathrm{d}z'\varphi_1(z) - \\ & - \left[\int_{-\infty}^{+\infty} \frac{\Lambda}{2\eta}R_i(z')z'\phi_2(z')\mathrm{d}z' + \int_{-\infty}^{+\infty} \frac{\Lambda^2}{2}R_r(z')z'^2\varphi_2(z')\mathrm{d}z'\right]\varphi_2(z) + \\ & + \left[\int_{-\infty}^{+\infty} \frac{\Lambda}{2\eta}R_i(z')\phi_2(z')\mathrm{d}z' + \int_{-\infty}^{+\infty} \Lambda^2 R_r(z')z'\varphi_2(z')\mathrm{d}z'\right] z\varphi_2(z) \end{aligned} \tag{26}$$

and

$$\begin{aligned} v_2 \;=\; & -\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \frac{1}{2\eta^2\lambda}(\sin\beta)R_r(z')\varphi^*(z',k)\phi(z,k)\mathrm{d}z'\mathrm{d}k + \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \frac{1}{2\eta^2\lambda}\{1-\cos\beta\}\times \\ & \times R_i(z')\phi^*(z',k)\phi(z,k)\mathrm{d}z'\mathrm{d}k + \left[\int_{-\infty}^{+\infty} \frac{\Lambda}{2\eta}R_r(z')z'\varphi_1(z')\mathrm{d}z' + \right. \\ & \left. + \int_{-\infty}^{+\infty} \frac{\Lambda^2}{2}R_i(z')z'^2\phi_1(z')\mathrm{d}z'\right]\phi_1(z) - \left[\int_{-\infty}^{+\infty} \frac{\Lambda}{2\eta}R_r(z')\varphi_1(z')\mathrm{d}z' + \right. \\ & \left. + \int_{-\infty}^{+\infty} \Lambda^2 R_i(z')z'\phi_1(z')\mathrm{d}z'\right] z\phi_1(z) + \int_{-\infty}^{+\infty} \frac{\Lambda}{2\eta}R_r(z')z'\varphi_2(z')\mathrm{d}z'\phi_2(z), \end{aligned} \tag{27}$$

where $\beta = 2\eta^2\Lambda\lambda(t - \frac{(z'-z)}{2\eta})$ is defined. It is worth pointing out that, in addition to a continuous mode, localized modes turn out to appear in the solution, which is essentially different from the case of the envelope soliton of the integrable cubic NLSE with a first-order temporal derivative [15]. Very recently, the localized modes in perturbed soliton bearing systems are revealed and generally thought to be intrinsic for nonintegrable models, for instance, the ϕ^4 model [16]. Returning to the restriction conditions imposed on the solution, we indicate that they can be satisfied by the extra freedoms we introduce in advance. In fact, they result in a sequence of novel equations:

$$\eta_{t_1} = \frac{\Lambda}{2}\int_{-\infty}^{+\infty} \mathrm{Re}(P\,e^{i\theta})\mathrm{sech}z\mathrm{d}z, \tag{28}$$

$$\zeta_{t_1} = -\frac{\Lambda}{2}\int_{-\infty}^{+\infty} \mathrm{Im}(P\,e^{i\theta})\tanh z\mathrm{sech}z\mathrm{d}z, \tag{29}$$

$$4\eta^2(\Lambda^{-2} - \frac{4}{3}\eta^2)\chi'_{t_1} = \int_{-\infty}^{+\infty} \mathrm{Re}(Pe^{i\theta})z\mathrm{sech}z\mathrm{d}z - 2\eta\Lambda\int_{-\infty}^{+\infty} \mathrm{Im}(Pe^{i\theta})z\tanh z\mathrm{sech}z\mathrm{d}z, \tag{30}$$

and

$$2\eta(\Lambda^{-1} - 4\eta^2\Lambda) \times (\theta_{1t_1} + 2\eta\Lambda^{-1}K\chi'_{t_1}) = \int_{-\infty}^{+\infty} \mathrm{Im}(Pe^{i\theta})\mathrm{sech}z(1 - z\tanh z)\mathrm{d}z - -2\eta\Lambda\int_{-\infty}^{+\infty} \mathrm{Re}(Pe^{i\theta})z\mathrm{sech}z\mathrm{d}z, \quad (31)$$

which govern the dynamical evolution of the bright soliton in the time. Following the usual definition of the width in the soliton theory, we define the width of the waveshape of a bright soliton as $w = 1/2\eta(2\zeta + b)$, and then we derive a useful equation:

$$w_{t_1} = 4\eta^2 w^3 \int_{-\infty}^{+\infty} \mathrm{Im}(P\,e^{i\theta})\tanh z\mathrm{sech}z\mathrm{d}z - w^2\int_{-\infty}^{+\infty} \mathrm{Re}(P\,e^{i\theta})\mathrm{sech}z\mathrm{d}z. \quad (32)$$

Now, we generate some results for two physical cases. At first, we show the consistent aspect of our theory with previous ones by considering the linear loss that is given by $P[u] = -\alpha_1 u$. This perturbation leads to $\mathrm{Re}(Pe^{i\theta}) = -2\eta\alpha_1\mathrm{sech}z$, and then $\eta_{t_1} = -2\alpha_1\eta\Lambda$ is derived from eq. (28). Thus, we obtain $\eta = \eta_0 e^{-2\alpha_1\Lambda t_1} = \eta_0 e^{-2\varepsilon\alpha_1\Lambda t}$ by integration. In this case, Λ remains constant, indicating that the propagation distance of the central point of the soliton can be calculated by $x = \Lambda t$. As a result, we can write

$$\eta = \eta_0 e^{-2\varepsilon\alpha_1 x}, \quad (33)$$

which recovers the well-known result in previous theories [1]. Secondly, we study the perturbation $P[u] = -i\alpha_2 u\partial|u|^2/\partial t$ accounting for the Raman effect. Using eq. (7), we derive $\mathrm{Im}(Pe^{i\theta}) = -32\eta^4\alpha_2\tanh z\,\mathrm{sech}^3 z$, which shows that the soliton's width and velocity are influenced. By eq. (32), we get $w_{t_1} = -8\alpha_2(2\eta)^6 w^3/15$; integrating this equation yields

$$w = w_0\left[1 + \frac{16}{15}\alpha_2(2\eta)^6 w_0^2 t_1\right]^{-\frac{1}{2}}, \quad (34)$$

which shows that the soliton is narrowed under this effect. In addition, we derive that the velocity decreases, obeying

$$\Lambda = \Lambda_0\left[1 + \frac{16}{15}\alpha_2(2\eta)^4\Lambda_0^2 t_1\right]^{-\frac{1}{2}}. \quad (35)$$

Here, we should note that, under the picture of waveform, the width is depicted differently, and the velocity of dynamical sense cannot be defined. Thus, eq. (34) and eq. (35) cannot be derived in previous theories.

In conclusion, we think that the waveshape and the waveform should be of the same importance for the description of a bright soliton in a fiber. However, the waveshape presents a more transparent picture of directly physical significance than the waveform, especially in the study of the bright soliton under perturbations. Hence, we believe that our theory is necessary and nontrivial. Moreover, the theoretical framework in this paper is distinct and normal, its idea is helpful for the study of other soliton problems as well.

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REFERENCES

[1] Agrawal G. P., *Nonlinear Fiber Optics* (San Diego, Academic Press, Inc.) 1989.
[2] Haus H. A. and Wong W. S., *Rev. Mod. Phys.*, **68** (1996) 423.
[3] Kivshar Yu. S. and Luther-Davies B., *Phys. Rep.*, **298** (1998) 81.
[4] Diament P., *Wave Transmission and Fiber Optics* (New York, Macmillan Publishing Co.) 1990.
[5] Elgin J. N., *Phys. Rev. A*, **47** (1993) 4331.
[6] Kivshar Yu. S. and Malomed B. A., *Rev. Mod. Phys.*, **61** (1989) 763.
[7] Kaup D. J., *Phys. Rev. A*, **42** (1990) 5689; **44** (1991) 4582.
[8] Burtsev S. and Kaup D. J., *J. Opt. Soc. Am. B*, **14** (1997) 627.
[9] Lamb G. L., *Element of Soliton Theory* (Wiley, New York) 1980.
[10] Scharf R. and Bishop A. R., *Phys. Rev. E*, **47** (1993) 1375.
[11] McLaughlin D. W. and Scott A. C., *Phys. Rev. A*, **18** (1978) 1652.
[12] Holmes M. H., *Introduction to Perturbation Methods* (Springer-Verlag, New York, Inc.) 1995.
[13] Tang Y. and Wang W., *Phys. Rev. E*, **62** (2000) 8842.
[14] Leung P. T., Suen W. M., Sun C. P. and Young K., *Phys. Rev. E*, **57** (1998) 6101.
[15] Keener J. P. and McLaughlin D. W., *Phys. Rev. A*, **16** (1977) 777.
[16] Kivshar Yu. S., Pelinovsky D. E., Cretegny T. and Peyrard M., *Phys. Rev. Lett.*, **80** (1998) 5032; Quintero N. R., Sánchez A. and Mertens F. G., *Phys. Rev. Lett.*, **84** (2000) 871.