

Resonance-enhanced signal detection and transduction in the Hodgkin-Huxley neuronal systems

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The ability of signal detection and transduction of the Hodgkin–Huxley neuronal systems, associated with rhythmic oscillations in the presence of external modulations, is studied. Both inhibitory and excitatory modulations, regarded as the total effects of the environment in which the neurons are located, are able to modulate the frequencies of the rhythmic oscillations of the neurons. Either subthreshold or suprathreshold rhythmic oscillations can provide the neural system with an effect of frequency selection in processing external signal. Resonance among the noise, the noise-induced oscillation, and the signal enhances intensively the capability of the neurons in processing the weak signal, especially when frequency of the signal is around that of the noise-induced rhythmic oscillation. Thus, the neuronal system can be adjusted to an optimal sensitive state for signal processing through the environmental modulations.

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I. INTRODUCTION

Attempts to understand how individual neurons or neuronal networks process information have occupied scientists for decades. In the neural systems, neurons are always located in an excitatory or inhibitory environment which attributes to various inputs, mainly the synaptic current, to the neurons. Many factors, such as fluctuations of synaptic input, dendritic or soma membrane parameters, and so on, make the environment noisy. All these effects together can be regarded as an external modulation of the environment to the neurons. It is suggested that the neurons can utilize such external modulation to process the input signals effectively [1,2], and the information processing is done under such conditions.

Recently, a phenomenon of stochastic resonance (SR) in neural systems has been extensively studied both experimentally [3–6] and theoretically [7–10]. By this nonlinear effect, the responses of neurons to a weak periodic signal can be optimized by suitable noise. The occurrence of the SR has been argued to be a mechanism for weak signal detection and transduction. In addition, the firing precision in response to a subthreshold stimulus can also be maximized by a noise with suitable intensity [9].

More interestingly, it has recently been demonstrated that a subthreshold oscillation originating from the intrinsic characteristic of neurons, in combination with external modulation, can provide the neurons with particular encoding properties [11] and also can enhance the ability of the neuron in signal detection and transduction [1]. Especially, a phenomenon of frequency sensitivity in weak signal detection has been examined both experimentally [12] and theoretically [13,14]. It was found that there exists a frequency range of 30–100 Hz in which the signal detection and transduction of neurons are more effective [12]. That is, the coherence of the spiking responses of neurons to those signals is enhanced, improving the ability of the neurons to detect weak signals. Such a frequency sensitivity was argued to result from a resonance between the intrinsic oscillation and the signal [12–14].

The studies mentioned above pointed out the importance of the intrinsic subthreshold oscillation in information pro-

cessing. In the presence or absence of the noise, the neuron, as a nonlinear oscillating system, can utilize the nonlinear resonance between the external signal and intrinsic oscillation to optimally detect, transduce or encode the signals with frequencies and strengths in a certain range. Resonance may be the most economic way for processing the input signals. In addition, it is well known that in the peripheral or central nervous system spontaneous rhythmic oscillations are ubiquitous phenomena [11]. How these rhythmic oscillations are generated and what roles these rhythmic oscillations play in information processing are still unclear. Do these rhythms relate to the intrinsic oscillations of neurons? How does a neuron (serving as a basic unit or informational processor, which possesses the ability of self-adapting or self-adjustment) use a suitable external modulation to adjust its rhythmic oscillation for effectively processing the input signals? Obviously, these questions are important and interesting for the understanding on the mechanism underlying the information processing.

To address these questions, we make a study based on a popular Hodgkin–Huxley (HH) neuronal model and a globally coupled network. By numerical simulations we find that different environments (excitatory or inhibitory), in which the neurons are located, together with the intrinsic characteristic of the neuron, can modulate a neuron or a network to generate rhythmic oscillations. Such a rhythm provides the neural system with an effect of frequency preference to external signals. In response to a weak signal, a resonance among the noise, the noise-induced oscillation, and the signal enhance intensively the ability of the neural system in detection and transduction of the external signal, especially when the frequency of the signal is around that of the rhythmic oscillation of the neurons. The frequency of the rhythmic oscillation can be adjusted by the modulations, so that the neurons can effectively process signals with various frequencies. This is of significant biological meaning.

This paper is organized as follows. In Sec. II the single HH neuronal model is described. The results and discussion for the single neuron case are presented in Sec. III, while those for the neuronal network case are given in Sec. IV. Finally, a conclusion is given in Sec. V.

II. MODEL

Let us start by considering the HH neuronal model. The HH neuronal model is a useful paradigm that accounts naturally for both the spiking behavior and refractory properties of real neurons [15], which is described by four nonlinear coupled equations: one for the membrane potential V and the other three for the gating variables: m , n , and h ; that is,

$$\frac{dV}{dt} = (I_{\text{ext}}(t) + I_{\text{ion}}(t))/C_m, \quad (1)$$

$$\frac{dm}{dt} = (m_{\infty}(V) - m)/\tau_m(V), \quad (2)$$

$$\frac{dh}{dt} = (h_{\infty}(V) - h)/\tau_h(V), \quad (3)$$

$$\frac{dn}{dt} = (n_{\infty}(V) - n)/\tau_n(V), \quad (4)$$

with

$$I_{\text{ion}}(t) = -g_{na}m^3h(V - V_{na}) - g_kn^4(V - V_k) - g_l(V - V_l), \quad (5)$$

$$I_{\text{ext}}(t) = I_0 + I_1 \sin(2\pi f_s t). \quad (6)$$

The ionic current $I_{\text{ion}}(t)$ includes the usual sodium (I_{na}), potassium (I_k), and leak (I_l) currents. The parameters g_{na} , g_k and g_l are the maximal conductances for the ions and leakage channels, and V_{na} , V_k , V_l are the corresponding reversal potentials. m_{∞} , h_{∞} , n_{∞} and τ_m , τ_h , τ_n represent the saturated values and the relaxation times of the gating variables, respectively. Detailed values of parameters can be found in Refs. [15,16]. $I_{\text{ext}}(t)$ is the total external stimulus received by the neuron. $I_1 \sin(2\pi f_s t)$ is a periodic signal with I_1 and f_s being the amplitude and frequency of the signal, respectively.

I_0 is a constant stimulus and is regarded as the simplest modulation to the neuron. A neuron always receives synaptic inputs from other neurons in the network, and there are also various fluctuations in its membrane. It is known that there are thousands of neurons connected to each other in the network. Thus, the average synaptic input to a neuron may vary slowly with time. For a simple case, the total inputs from the environment could be assumed as a constant stimulus, I_0 . Here, the value of I_0 may be positive, i.e., excitatory, which can depolarize the postsynaptic membrane and increase the probability of the firing. Alternatively, the value of I_0 may be negative, i.e., inhibitory, which can hyperpolarize the postsynaptic membrane and thereby reduce the possibility of the firing.

III. RESULTS FOR A SINGLE HH NEURON

In this section, we examine how the intrinsic subthreshold oscillation is influenced by the external modulation (see Sec. III A), and how the intrinsic subthreshold oscillation affects the detection capability of the neuron to the periodic signal

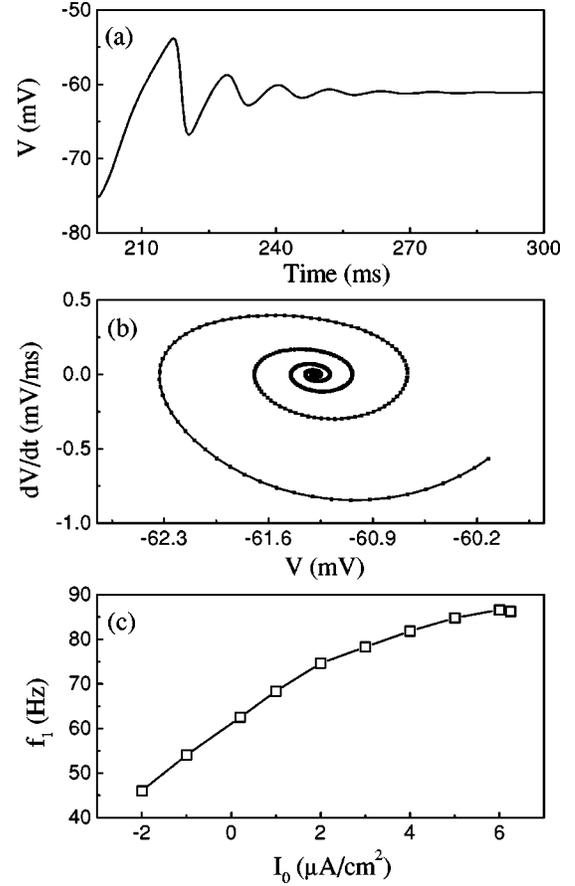


FIG. 1. The potential $V(t)$ of a neuron varying with time $I_0 = 5 \text{ A/cm}^2$. (b) Phase plot dV/dt versus V with $I_0 = 5$ in which the trajectory moves clockwise around the loop, and the equilibrium state is a global attractor. (c) The frequency f_1 of subthreshold oscillation of one neuron versus input I_0 (I_0 increases from -2 to 6).

(see Sec. III B). We demonstrate that the inhibitory or excitatory modulation can set the neuron to a suitable sensitive state for detecting the input signals with various frequencies. In the presence of noise, we investigate how the neuron, without external signal, produces coherent oscillations. The effect of external modulation is also studied (see Sec. III C). It is found that the noise-induced oscillations provide the neuron with an effect of frequency preference to the external signal. In the Sec. III D, we study mainly the effect of the frequency preference and the SR on the detection and transduction of the external weak periodic signal.

A. Deterministic case without periodic input

The HH neuron is a self-excitable system. For a DC input I_0 , the firing threshold is $I_c \approx 6.2 \text{ } \mu\text{A/cm}^2$ [16]. [Hereafter, we do not give out the unit for all the currents, e.g., for I_0 , I_1 , and the current appearing later.] When $I_0 < I_c$, the membrane exhibits a damped subthreshold oscillation [see Fig. 1(a)] due to the fact that the stable fixed point is the global attractor of the system [see Fig. 1(b)]. The birth of the limit cycle, i.e., the firing of spikes, occurs at I_c owing to the saddle-node bifurcation. The subthreshold oscillation result-

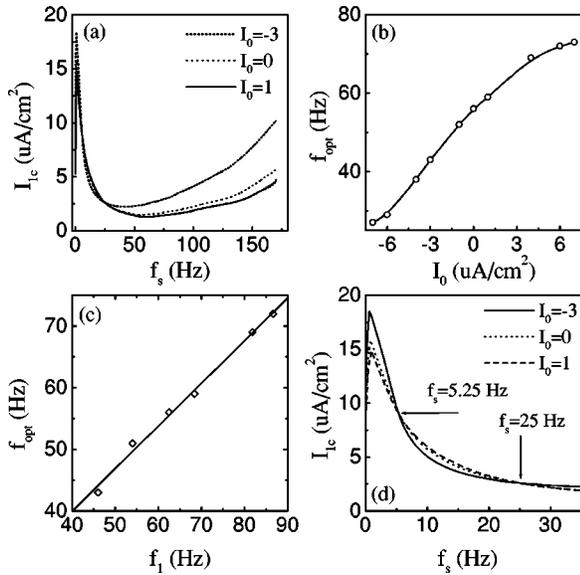


FIG. 2. The firing onset curve of the HH neuron in the absence of noise: (a) The threshold amplitude I_{1c} versus input signal frequency f_s for $I_0 = -3, 0$, and 1 , respectively. (b) The optimally resonant frequency f_{opt} versus different value of I_0 . (c) Correlation between the optimal resonant frequency f_{opt} and the frequency f_1 of the subthreshold oscillation for I_0 varying from -2 to 6 . (d) Details of I_{1c} versus f_s for $0 < f_s < 35$ Hz. There are two crossovers dividing the firing onset curves into three frequency regions.

ing from the excitability inherent in the neuron is considered to be intrinsic, and its frequency rises monotonically from 45 to 85 Hz as I_0 increases from -2 to 6 [see Fig. 1(c)]. This indicates that there exists a dependence of the frequency of the intrinsic oscillation on I_0 . In the following, we shall examine the role of the intrinsic oscillation playing in the signal processing.

B. Deterministic case with periodic input

Now we investigate the dynamical responses of a HH neuron to a periodic signal [17,18] in both cases of excitatory (e.g., $I_0 > 0$) and inhibitory (e.g., $I_0 < 0$) modulation. First, we consider the dependence of the critical amplitude I_{1c} of the signal on its frequency f_s . Figure 2(a) shows the frequency dependence of I_{1c} on f_s for different values of I_0 . Here all values of I_0 are smaller than its critical value, i.e., $I_0 < I_c$. The curves are nonmonotonic and give out the boundary between the nonfiring and the firing states. It is noted that in these three curves there exists a frequency range of 30–100 Hz, i.e., a sensitivity frequency band for the signal detection, where a lower signal strength I_1 can evoke the neuron to fire. When the frequency of the signal is very low ($f_s < 0.7$ Hz) or very high ($f_s > 100$ Hz), the threshold I_{1c} increases monotonically as the value of f_s increases. This is similar to the case in the bistable system, which is due to the hysteresis of the nonlinear response of the system to the periodic input [19]. In the intermediate range ($0.7 < f_s < 100$ Hz), the threshold I_{1c} first decreases and then increases, and there is a minimum of I_{1c} within 30–80 Hz.

Physically, the existence of such a frequency range is due to the nonlinear resonance between the aforementioned intrinsic oscillation and the periodic signal [14]. When both frequencies are matchable, the input signal can most effectively transfer energy to the neuron to evoke the firing of spikes. Thus, less stimulus strength is needed for the neuron to fire. For each I_0 , there is an optimally resonant frequency f_{opt} at which a maximal resonance occurs and a minimal I_{1c} is obtained. It is worth noting that the frequency range shifts to high frequency as I_0 increases. Figure 2(b) shows the relation of f_{opt} against I_0 . We find that the optimally resonant frequencies are in a range of 30–80 Hz when I_0 varies from -6 to 6 , in a range of physiological significance. Obviously, the change of the optimally resonant frequency with I_0 is due to the fact that the frequency of the intrinsic subthreshold oscillation varies with I_0 . This can be seen clearly in Fig. 2(c). For I_0 in the range from -2 to 6 , the correlation between the frequency f_1 of the subthreshold oscillation [see Fig. 1(c)] and the optimally resonant frequency f_{opt} is shown. The perfect linear correlation (with correlation coefficient $R > 0.99$) reflects the consistency of both frequencies varying with I_0 . This implies that the subthreshold oscillatory behavior of the neuron provides itself with an effect of frequency selection to input signal via resonance. Since I_0 represents the total effects of the environment, the dependence of f_{opt} on I_0 suggests that the environment in which neurons are located can modulate the neurons to a suitable sensitive state for detecting the input signals with various frequencies. That is, the signal processing may be carried out with a minimum expenditure of energy by a maximal resonance between the intrinsic oscillation and the signal if the modulation can be properly controlled by changing the value of I_0 . In the neural system, such modulations may be performed by various neurotransmitters, neuromodulators, synaptic connection, or background oscillations, etc. [2].

Furthermore, it is noted that in Fig. 2(d) there are roughly three frequency ranges which are divided by two crossovers, i.e., $f_s < 5.25$ Hz, $5.25 < f_s < 25$ Hz, and $f_s > 25$ Hz, respectively. In the region of $5.25 < f_s < 25$ Hz, the inhibitory modulation (e.g., $I_0 = -3$) is better than the excitatory one (e.g., $I_0 = 1$) for the neuron to detect the input signals. On the contrary, in the other two regions, the excitatory modulation is more effective than the inhibitory one to enhance the ability of the neurons for the detection of the input signals. This results from the difference of the optimally resonant frequency region and may have significant effects on the signal detection. Thus, the occurrence of different rhythmic oscillations for the neurons may be due to their location in different modulatory environments, which needs further confirmation. All these suggest that different modulations, together with the intrinsic oscillation, may result in an effective mechanism of frequency selection by which the neurons can detect and transduce input signals effectively [1,13,14].

C. Stochastic case without periodic input

In order to examine the ability of the signal transduction of the neurons in more realistic cases of different modulations, we add a noisy current I_{noise} to the neuron in two cases

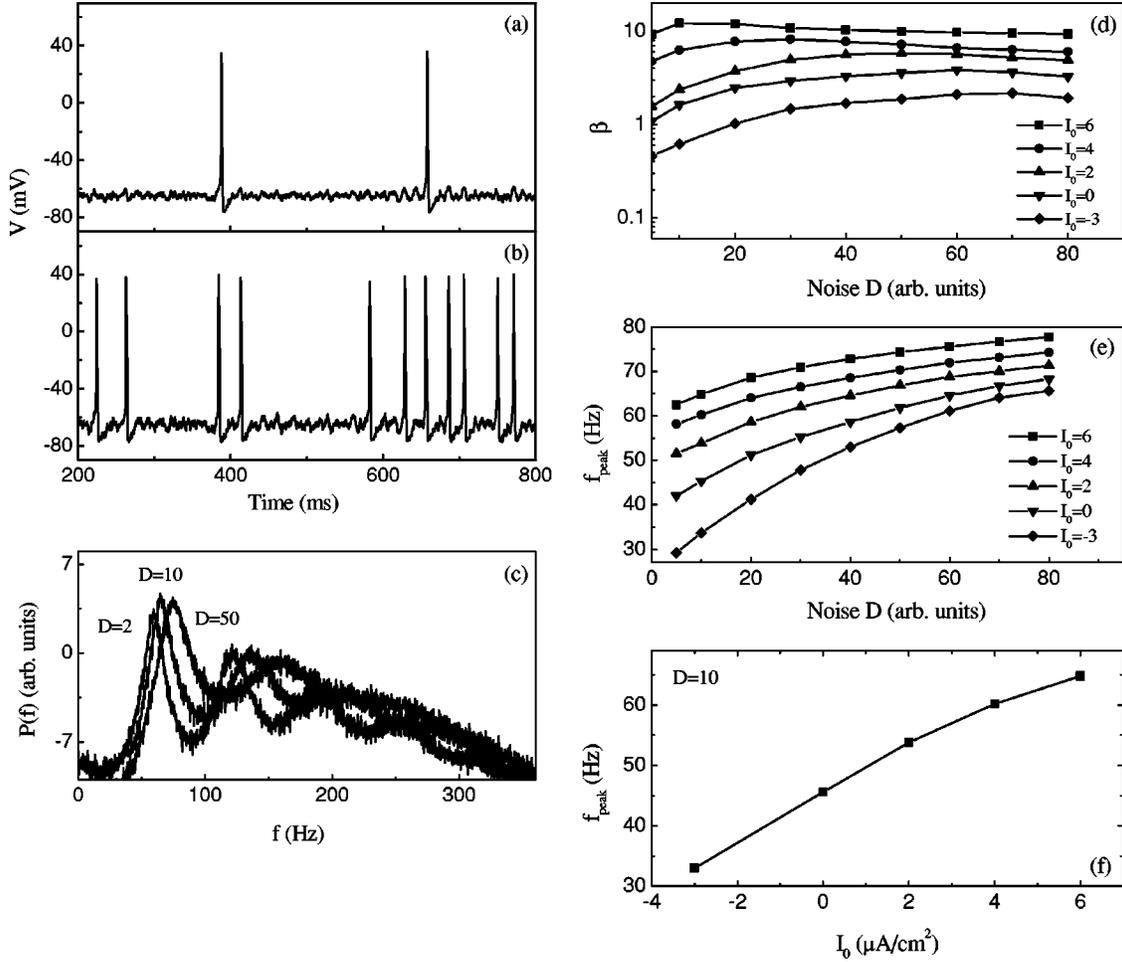


FIG. 3. For $I_0=0$, the membrane potential $V(t)$ of a neuron varying with time for a noise with $D=1$ (a), and $D=5$ (b). (c) The PSD (in \log_{10} scale), represented by $P(f)$, of the noise-induced firings for three noise intensities in the case of $I_0=6$. (d) The measure of coherence β versus noise intensity D for different I_0 . (e) The frequency f_{peak} of the main peak in the PSD versus D for different I_0 . (f) For $D=10$, the frequency f_{peak} of the main peak in the PSD versus the current I_0 .

of $I_0 = -3$ and 1, respectively. Here, I_{noise} represents the noisy component of the stimulus to a neuron from external fluctuations or intrinsic fluctuations of the neuron itself. We model this noisy current as an additive noise from an Ornstein–Uhlenbeck (OU) process,

$$\tau_d \frac{dI_{\text{noise}}}{dt} = -I_{\text{noise}} + \sqrt{2D}\xi(t), \quad (7)$$

where $\xi(t)$ is Gaussian white noise, and D and τ_d are the noise intensity and the correlation time of the OU noise. In the following numerical simulations, we let $\tau_d = 2$ ms. Thus $I_{\text{ext}}(t)$ is now set as

$$I_{\text{ext}}(t) = I_0 + I_{\text{noise}}. \quad (8)$$

Here, I_0 together with I_{noise} is the external modulation. This may be a more realistic consideration of the external modulation to the neuron. Due to the introduction of the noise, the dynamic equations for the neuron become stochastic. In the following, the numerical integration of stochastic HH equations is done by using second-order stochastic Runge–Kutta

algorithms suggested in Ref. [20] to obtain a required accuracy, and the time step is taken as 0.02 ms.

The weak noise makes the membrane potential fluctuate near the firing threshold and display sustaining subthreshold oscillation [see Fig. 3(a)]. Occasionally, spontaneous spikes occur due to the noise-induced threshold crossing. When the noise intensity becomes slightly large, noise-induced firings [see Fig. 3(b)] around a main frequency occur which may relate to a so-called coherence resonance (CR) [21] and has been discussed in detail in Ref. [22]. Here, the main frequency denotes the highest peak of the PSD [see Fig. 3(c)], reflecting the most intervals of the firings. Figure 3(c) shows the PSD, represented by $P(f)$, for three different noise intensities $D=2$, 10, and 50, respectively, for $I_0=6$. The maximal coherence of the system is around a noise with $D=10$ (see the following discussion).

We can characterize the CR quantitatively via a coherence factor β [22], which is a measure of coherence and defined as

$$\beta = hf_{\text{peak}} / \Delta f_{\text{peak}}, \quad (9)$$

where h and f_{peak} are the height and the frequency of the highest peak in the PSD, and Δf_{peak} is the width of the peak at its half maximal height. Thus, for various values of I_0 , the occurrence of CR can be seen clearly from the dependence of β on D [see Fig. 3(d)]. It is clear that the coherence becomes strong as the value of I_0 increases. From Fig. 3(d), we note that for each I_0 , there exists an optimal noise intensity D with which the value of β is maximal, especially for the cases of large I_0 . (When $I_0 \leq -3$, the CR is inconspicuous.) Similar to the mechanism of CR for neurons near the bifurcation point [22], there exist two time scales in the system, one is the period T_0 of oscillations of the limit cycle (the firing), the second is the activation time T_α , i.e., the mean time to drive the system from its stable point to the limit cycle region. For a fixed value of I_0 , when the noise intensity increases to an optimal level, the noise-induced activation time reaches a value which is matchable with T_0 , leading to the maximal coherent motion of the neuron, i.e., the manifestation of the CR [22]. When I_0 decreases, i.e., setting the system far from the bifurcation point, a large value of the noise intensity is needed to decrease T_α so as to obtain a matching with T_0 . That is, for a small I_0 , one needs a large optimal noise intensity to obtain a maximal coherence of the system. This can be seen from the results shown in Fig. 3(d).

Since the CR phenomenon is also found in the systems without the intrinsic oscillation, e.g., the leaky integrate-and-firing neuron [23], in this work firings around a main frequency may be the properties of the CR in the HH neurons, not the general CR. To see this clearly, let us study the factors affecting the frequency characteristic of the noise-induced oscillations of the stochastic HH neuron. Noted that the CR in the HH neuron near the bifurcation point is treated in Ref. [22]. Differently, the HH neuron treated here is far from its bifurcation point. As a result, the time scale of noise-induced oscillation depends largely on the noise intensity D [see Figs. 3(a) and 3(b)]. From Fig. 3(e), we note that the frequencies of the noise-induced oscillations depend mainly on both the noise I_{noise} and the modulation I_0 . When the noise intensity D varies from 5 to 80, for $I_0 = -3$, f_{peak} in the PSD increases from 30 to 66 Hz, while for $I_0 = 6$, f_{peak} of the PSD increases from 62 to 78 Hz. Nevertheless, when D is set out of this range, the main peak will be much low and broad. For each fixed noise intensity, there is a monotonic increasing of the peak frequency f_{peak} with the increasing of I_0 , for example, see Fig. 3(f), which shows a relation of f_{peak} varying with I_0 for $D = 10$. As a result, the frequency characteristic of the noise-induced oscillation indeed originate from the intrinsic oscillatory behavior of the HH neuron, as the reason for f_{opt} (see Fig. 2), and the frequency of the firings induced by the CR is close to the subthreshold intrinsic frequency. (Recently, there is a detailed discussion about the noise-induced eigenfrequency for different cases of a stochastic FitzHugh–Nagumo model systems [24].)

Therefore, either subthreshold or superthreshold oscillations can be regarded as the manifestation of the intrinsic oscillations. These oscillations have been observed extensively in experiments in various neuronal types, such as in neocortical neurons [25] and thalamic neurons [26]. The fluctuating modulation, together with the inherent bistability

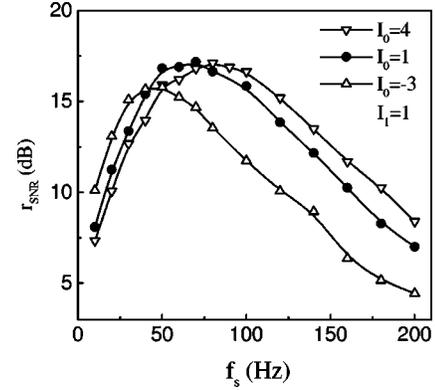


FIG. 4. The SNR, represented by r_{SNR} , of a neuron versus input signal frequency f_s for $I_0 = 4, 1, \text{ and } -3$ in the case of $D = 5$, respectively.

of the neuron, plays an important role in generating these oscillations. It was suggested that these oscillations provide an internal time clock for neural information detection, transduction, and encoding, and even supply a mechanism underlying synchronization and binding function for neuronal activities [11,27].

D. Stochastic case with periodic input

From the above discussion, due to the CR effect, a noisy HH neuron can be viewed as a spontaneous rhythmic oscillator. It is interesting to examine what happens when an external weak signal $I_1 \sin(2\pi f_s t)$ (I_1 is set as $1 \mu\text{A}/\text{cm}^2$ to make the signal be subthreshold) is input to such an oscillator.

Figure 4 shows the relationship of the signal-to-noise ratio (SNR) [28], represented by r_{SNR} , versus the frequencies of the weak periodic signals for different values of I_0 in the case of a fixed noise intensity $D = 5$. It can be seen clearly that the SNRs for the frequencies located in 20–100 Hz have large values for all curves. This indicates that the neuron is more sensitive to the signals with frequencies in this range. That is, the ability of the neurons in processing the signals is significantly improved in the presence of a weak noise, especially as the frequency of the signal is in the sensitivity region.

Generally, the SR is an effect which describes the coincidence of two time scales in a periodically modulated multistable, noisy nonlinear system. One time scale is the period of the periodic forcing or signal and the other relates to the well-to-well switching rate induced by the noise. The optimal enhancement effect on the switching is obtained at a suitable noise level which produces a maximum cooperative between the noise-induced transitions and the periodic signal. Now, for a noisy HH neuron, due to the intrinsic oscillation, the noise-induced threshold crossing rate is around a main frequency related to the natural frequency of the neuron. Thus, when an external signal is input to such an oscillator, the time scale related to the noise is determined not by a general threshold transition rate, but a rate around a main frequency of the noise-induced oscillation. When the frequency of a periodic signal is close to this rate, the coherence

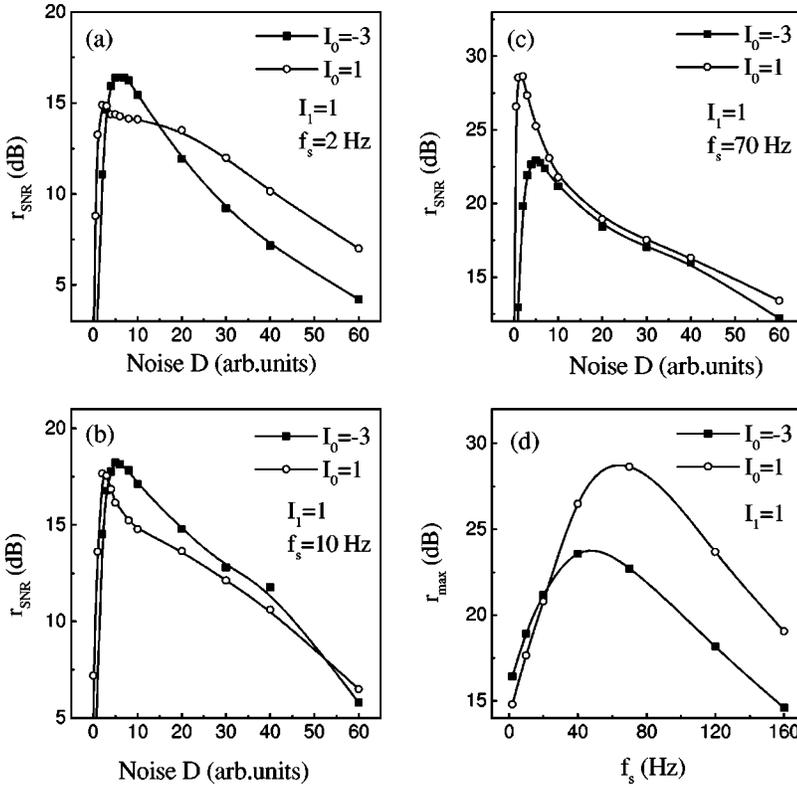


FIG. 5. (a)–(c) r_{SNR} of a neuron versus noise intensity D for various signal frequencies f_s in the case of $I_0 = -3$ and 1, respectively. (d) r_{max} versus frequency f_s of input signals for $I_0 = -3$ and 1, respectively. (For each f_s , r_{max} is obtained in different optimal noise intensities D .)

of the motion of the system will be strengthened largely by the resonance, through which the neuron fires spikes around the maxima of the signal and more energy of the environment is transferred to the output signal, leading to a large value of the SNR. The noise-induced oscillation provides an effect of frequency preference to external signal for the neurons in a noisy environment. This is the physical reason for the existence of the frequency sensitivity range of 20–100 Hz for the periodic signal. Indeed, such a resonance is a cooperative effect among the noise, the noise-induced oscillation and the external signal, which is related to the SR. The conditions for generating such a resonance include three factors: the noise, an intrinsic oscillatory behavior of the nonlinear system, and a weak periodic signal.

In addition, from Fig. 4, we note that, for each I_0 , there is a maximal value of SNR around its optimally resonant frequency. As I_0 increases, the whole frequency sensitive range (20–70 Hz for $I_0 = -3$) shifts to a slightly higher frequency region (30–100 Hz for $I_0 = 4$). This can be understood from the results shown in Fig. 3(e) and the above discussion. Different values of I_0 can adjust the intrinsic frequency characteristic of the noise-induced oscillations, and thus set the neuron to different sensitive states in signal processing.

It is easy to think that for each signal there will exist an optimal noise intensity which makes the resonance among the noise, the noise-induced oscillation, and the signal be maximized via SR. Figures 5(a)–5(c) show the SNR versus the noise intensity D for different frequencies of signals, e.g., $f_s = 2$, 10, and 70 Hz, respectively. We can see that all the curves clearly exhibit a typical characteristic of the SR: the value of SNR first sharply rises and then drops as the noise intensity increases. For each frequency, there exists an opti-

mal noise intensity D with which the neurons can obtain a maximal value of the SNR. Figure 5(d) presents the relationship of the maximal SNR, represented by r_{max} , versus the frequency f_s of the input signal in the cases of $I_0 = -3$ and $I_0 = 1$, respectively. From Fig. 5(d), we can see that there is a crossover around $f_s = 25$ Hz. When $f_s < 25$ Hz, we find that there always exists a suitable noise region with which the neurons can obtain a higher SNR for the inhibitory modulations (e.g., $I_0 = -3$) than that for the excitatory modulations (e.g., $I_0 = 1$). On the contrary, for the signals with high frequency $f_s > 25$ Hz, the excitatory modulations are more effective for neurons to obtain a high SNR. This crossover may result from the crossover in the firing onset curves shown in Fig. 2(d). Therefore, it is clear that in Fig. 5(d) the excitatory modulations are of great advantage for the neurons in detection and transduction of the signals with frequencies in a range of 30–70 Hz, the same as the γ frequency band. Differently, the inhibitory modulations have the advantage for the neurons in detection and transduction of signals with low frequencies. It is noted that the optimal noise intensity of these maximal SNRs are basically the same for different frequencies.

The above studies mainly deal with the cases of a single neuron with the noisy modulations. Although our consideration of both the excitatory and the inhibitory modulations includes the fluctuations of the environment, the real situation should be more complex and more detailed consideration is needed. However, we believe that the general physical picture is the same.

IV. RESULTS FOR A NETWORK

Now, we construct a globally coupled HH neuronal network and examine the effects of the excitatory or inhibitory

modulations on the ability of transduction to the periodic signals. The model for the network is presented in Sec. IV A. In the absence of the signal, a phenomenon of noise-induced synchronized rhythms is studied in Sec. IV B. We also study the effect of the excitatory and inhibitory connections on the features of those rhythms. As in the single neuronal case, these synchronized rhythms provide the network with an effect of frequency preference. Finally, in Sec. IV C, we study mainly the effect of the frequency preference and the resonance on the capability of the network in detection and transduction of the weak signal.

A. The network model

The dynamic equations for the network consisting of the HH neurons can be presented as follows [14,16]:

$$\frac{dV^i}{dt} = (I_{\text{ext}}^i(t) + I_{\text{ion}}^i(t)) / C_m, \quad (10)$$

$$i = 1, 2, \dots, N,$$

where the ionic current I_{ion}^i is the same as defined in the case of the single neuron, and $I_{\text{ext}}^i(t)$ is assumed as

$$I_{\text{ext}}^i(t) = I_0 + I_{\text{syn}}^i(t) + I_1 \sin(2\pi f_s t) + I_{\text{noise}}(t). \quad (11)$$

Here we let $I_0 = 0$ in order to study mainly the effect of $I_{\text{syn}}^i(t)$ on the neurons. As in the single neuron case, the summation of $I_{\text{syn}}^i(t)$ and $I_{\text{noise}}(t)$ are considered as an external modulation to the i th neuron in the network. The amplitude of the signal I_1 is set as $1 \mu\text{A}/\text{cm}^2$. The number of neurons in the network is taken as $N = 200$. The synaptic current $I_{\text{syn}}^i(t)$ is described as

$$I_{\text{syn}}^i(t) = -\frac{1}{N} \sum_{j=1, j \neq i}^N g_{\text{syn}} \alpha^i(t-t^j) \times (V^i - V_{\text{syn}}^{ij}), \quad (12)$$

with

$$\alpha^i(t) = (t/\tau) \exp[-t/\tau].$$

In Eq. (11) g_{syn} is a parameter of the synaptic conductance and the peak synaptic conductance is g_{syn}/e . V_{syn}^{ij} is the synaptic potential between the i th and the j th neurons. τ is the characteristic time of the synaptic interaction, which is set as 2 ms. t^j is the time when the interaction starts, i.e., the time of the firing of the presynaptic j th neuron.

Whether the synaptic effect is excitatory or inhibitory depends on the value of V_{syn}^{ij} . For the excitatory synapses $V_{\text{syn}}^{ij} = 0$ mV, we have $I_{\text{syn}} > 0$. Differently, for the inhibitory synapses $V_{\text{syn}}^{ij} = -80$ mV, we have $I_{\text{syn}} < 0$. In order to introduce the inhomogeneity in the network, we assume that some synapses are randomly excitatory and some randomly inhibitory. That is, we let V_{syn}^{ij} be randomly equal to either -80 or 0 mV. Thus, the average synaptic input to the neurons in the network is

$$\langle I_{\text{syn}}(t) \rangle = -\frac{1}{N^2} \sum_{i=1}^N \sum_{j=1, j \neq i}^N g_{\text{syn}} \alpha^i(t-t^j) \times (V^i - V_{\text{syn}}^{ij}). \quad (13)$$

Clearly, this average of the synaptic activities associates with the so-called local field potential (LFP) [29], which can be regarded as a modulation to each neuron in network as discussed in the above section. Furthermore, we consider the synaptic connection to be global, i.e., each neuron receives $N-1$ synaptic interactions (or couplings) from all other neurons in the nervous system. We define η as the percentage of the excitatory couplings in the total number of the synaptic interactions. Therefore, the resultant value of $\langle I_{\text{syn}}(t) \rangle$ is mainly affected by η .

The output of the network is defined as

$$V^{\text{out}}(t) = \frac{1}{N} \sum_{i=1}^N \theta(V^i(t) - V^*). \quad (14)$$

V^* is a membrane potential threshold (here, $V^* = -20$ mV), and $\theta(x) = 1$ if $x > 0$ and $\theta(x) = 0$ if $x < 0$. Here we consider the average of the firing activities of all neurons as the output of the network, which may encode the input signal.

B. Stochastic case without periodic input

First, we let $I_1 = 0$, i.e., in the absence of external signal, and study the dynamical features of the network in the presence of noise. Figures 6(a)–6(f) show the time evolution of the firings and the corresponding averaged synaptic current $\langle I_{\text{syn}}(t) \rangle$ with different noise intensities D in the case of $\eta = 50\%$ and $g_{\text{syn}} = 2 \text{ ms}/\text{cm}^2$. For $D = 0.5$, the firing phases of the neurons in the network are random in time and are irrelevant to each other [see Fig. 6(a)]. $\langle I_{\text{syn}}(t) \rangle$ is low and shows high randomness [see Fig. 6(d)]. However, the spatiotemporal order is optimized by a suitable noise with intensity $D = 5$ via a CR-like mechanism, and the coherent synchronized rhythmic oscillation is obtained [see Fig. 6(b)]. $\langle I_{\text{syn}}(t) \rangle$ has a large value and exhibits an apparent periodicity [see Fig. 6(e)]. When the noise intensity increases to a large value, e.g., $D = 40$, the stochastic nature of the noise dominates the dynamics of the network, and the synchronized firings are destroyed [see Fig. 6(c)]. The relevant $\langle I_{\text{syn}}(t) \rangle$ becomes low and random [see Fig. 6(f)]. Associated with the CR behavior in the single neuron case, when the noise is weak, the firings of individual neurons in the network are few and random in time, showing few correlations with each other. As the noise intensity increases, neurons generate self-sustained oscillations, which may act as an internal *clock* affecting the dynamics of the network. The coupling between the neurons plays an important role for the coherent oscillations. When the coupling strength is weak, e.g., $g_{\text{syn}} < 0.5 \text{ ms}/\text{cm}^2$, it is found that the firing phases of neurons in the network are all random in time and irrelevant to each other for various noise intensities. There is no coherent synchronized rhythmic oscillation. Only when the coupling strength be large, e.g., $g_{\text{syn}} \geq 2 \text{ ms}/\text{cm}^2$, the network shows spatiotemporal ordered firings since the neurons are

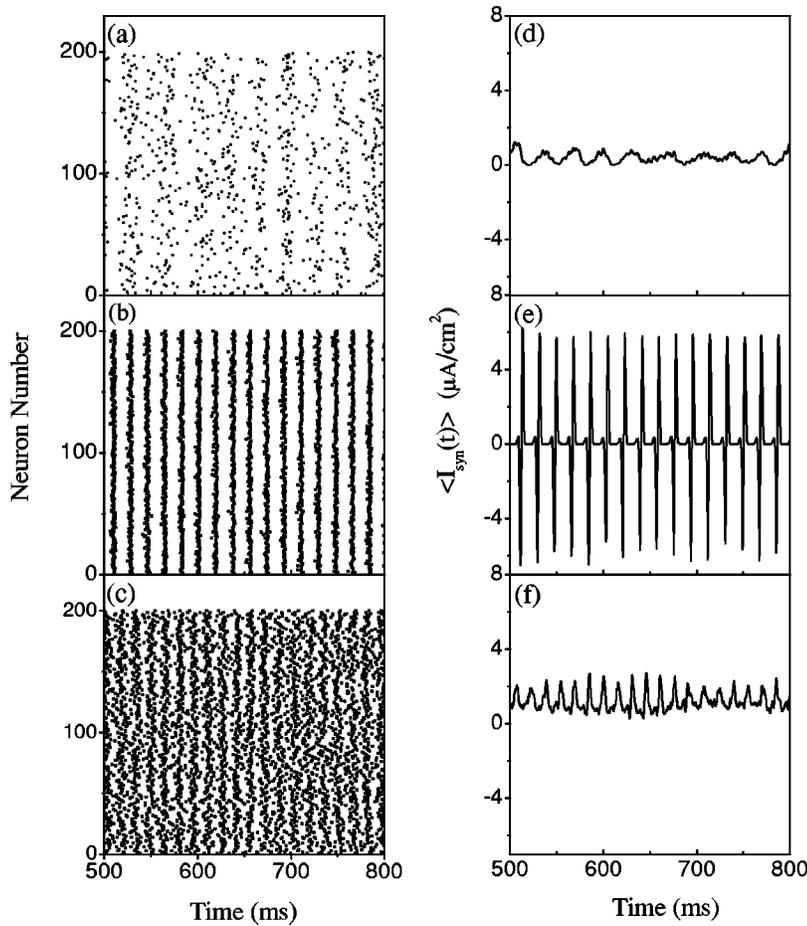


FIG. 6. The spatiotemporal firings and corresponding averaged synaptic current $\langle I_{syn}(t) \rangle$ of the network with $\eta=50\%$ and $g_{syn}=2$ ms/cm^2 for different noise intensities: $D=1$ [(a) and (d)], $D=5$ [(b) and (e)], and $D=40$ [(c) and (f)].

forced to fire in phase by the strong coupling. Such ordered firings are optimized by a suitable noise intensity, showing a CR-like behavior like the one shown in Fig. 6(b). In brief, the noise-induced spiking can evolve to a refined spatiotemporal order through the dynamical optimization among the autonomous oscillation of individual neurons, the coupling of the network, and the noise. This may be the mechanism underlying the CR-like phenomenon of the HH neuronal network. This phenomenon can be seen more clearly from the corresponding PSDs of the output of the network shown in Fig. 7. For a suitable noise with $D=5$ a sharp peak appears and is located at the frequency of the synchronized rhythmic oscillations (or firings) of the network. When the noise intensity increases, the peak becomes broad and low. A relevant measure of coherence β , i.e., Eq. (9), versus D is shown in the inset of Fig. 7. It is seen that β increases dramatically with D first, showing the onset of synchronization. Then a wide plateau for the value of β is followed, indicating that due to the strong coupling, the self-evolved synchronized rhythm is stable against a large range of noise intensities. Finally, β decreases quickly as the noise intensity D increases further.

Now, we investigate the effect of η on the dynamics of the network. With a same noise intensity $D=5$, Fig. 8(a) shows the PSDs (in logarithmic scale) of the output of the network for different η . It is seen that the frequencies of the rhythmic oscillations of the neurons vary from 40 to 60 Hz as the value of η increases. For each value of η , there is an

optimal noise intensity (or a small range) where the factor β has a maximal value. The dependence of β on the noise intensity D corresponding to the maximal coherence is of bell shape for each η (cf. the inset in Fig. 7), and there is a little difference between the optimal values of D corresponding to the maximal coherence of the network. The period of the noise-induced firings of the network seems to be characterized by the sum of the width of the output pulse and the refractory period, namely about 20 ms [cf. Fig. 3(b)], so the peak frequencies become close to each other about 50 Hz, which is indeed in a range of 40–60 Hz. The difference of

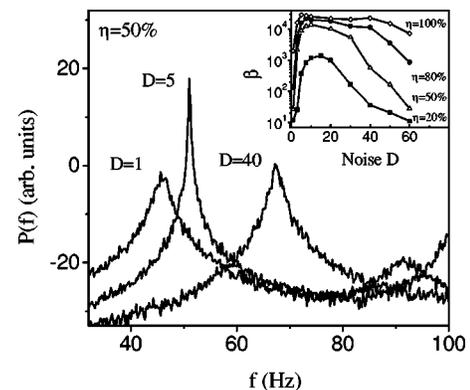


FIG. 7. Corresponding to Figs. 6(a), 6(b), and 6(c), the PSDs (in \log_{10} scale) of the output of the network. Inset: The measure of coherence β versus different noise intensities.

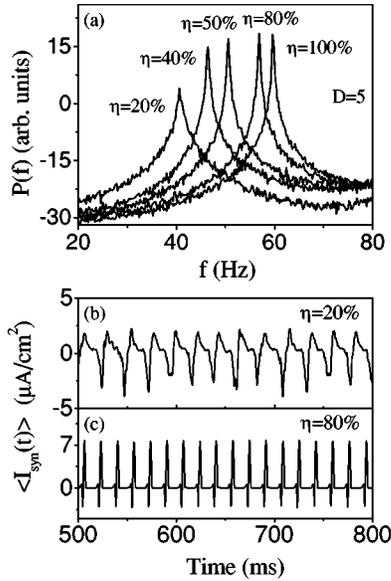


FIG. 8. (a) The PSD (in \log_{10} scale) of the output of the network with $g_{\text{syn}}=2$ ms/cm² and a fixed noise intensity $D=5$ for different ratio of excitatory to inhibitory couplings η . Averaged synaptic input $\langle I_{\text{syn}}(t) \rangle$ versus time in the case of $D=5$ and $g_{\text{syn}}=2$ ms/cm² for $\eta=20\%$ (b), and 80% (c).

frequencies of the network may be caused by the difference of the optimal noise intensity corresponding to the maximal coherence for different η . Figures 8(b) and 8(c) show the time evolution of $\langle I_{\text{syn}}(t) \rangle$ for two different η . The averaged synaptic currents in these two cases display clearly periodic features as that shown in Fig. 6(e). Except the periodic feature, the amount of the excitatory component [with positive value of $\langle I_{\text{syn}}(t) \rangle$], the peak value and the width of the pulses in $\langle I_{\text{syn}}(t) \rangle$ are different for different η . Different disturbances, whether positive or negative, of $\langle I_{\text{syn}}(t) \rangle$ result in different firing states of the neurons in the network, leading to the difference between the optimal values of D for various η . In addition, by observing the time series of $V^{\text{out}}(t)$ and $\langle I_{\text{syn}}(t) \rangle$ for $\eta=20\%$ simultaneously, it is found that the positive disturbance of $\langle I_{\text{syn}}(t) \rangle$ is effective for the firing. This is also for the case of $\eta>20\%$. However, the presence of negative disturbance may raise the firing probability of the neurons for the following positive one, as discussed in Ref. [30], which needs further study in our case.

Based on the discussion above, we conclude that the physical reason for the frequency characteristic of the network is due to the difference of the optimal noise intensities for different values of η . The modulation to the neurons in the network includes a pulslike synaptic component, which is the sum of alpha functions, i.e., Eq. (12). These are clearly related to the magnitude of η and the coupling g_{syn} which can adjust the frequency of the rhythmic firings of the network, as shown in Fig. 8(a).

C. Stochastic case with periodic input

The noise-induced synchronized rhythmic oscillation can be viewed as spontaneous rhythmic oscillation of the network in the noisy environment. It is interesting to explore the

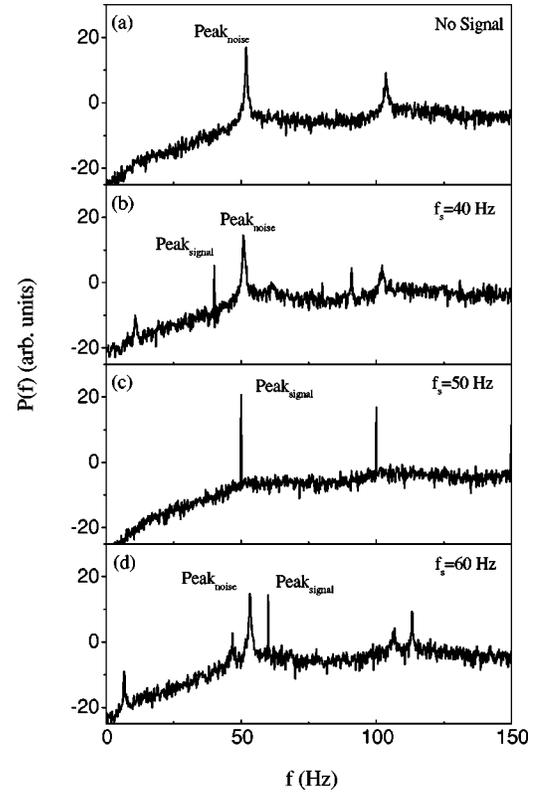


FIG. 9. (a) With the same condition in Fig. 8(a), the PSD of the spike trains of one neuron in the network for $\eta=50\%$. When input a signal to the network, the relevant PSD for $f_s=40$ Hz (b), 50 Hz (c), and 60 Hz (d).

role of this rhythmic oscillation playing in the information processing. From Fig. 8(a), we note that the frequency of rhythm of the network for $\eta=50\%$ is about 50 Hz, This also can be seen from the PSD of the spike trains of one neuron in the network for $\eta=50\%$ [see Fig. 9(a)]. Compared with the CR case in a single neuron, the peak in the PSD of a neuron in the network is much higher and narrower. This is due to the effect of recurrent synaptic current of the network. Under such a condition, when a periodic signal is input to the network, the corresponding PSDs of the spike trains of one neuron in the network are shown in Figs. 9(b)–9(d). We find that when the frequency of the signal is near 50 Hz, roughly within a range of $45 < f_s < 55$ Hz, the noise-induced peak becomes very small and even disappears, while the signal peak is intensively enhanced and locates tightly in the frequency of the signal. Without doubt, a nonlinear resonance occurs, which makes the system absorb intensively the energy of the environment (including the noise) and oscillate tunefully with the external periodic forcing. In such a case, the coherent motion of the neurons is maximized by the signal and the noise. Thus, the neurons in the network fire spikes with more strong coherence with the signal, leading to a high peak in the PSD. When the frequency of the signal is far away from that of the noise-induced rhythm of the network, such as for $f_s=40$ and 60 Hz, the two distinct time scales in the system one from the signal, and the other from the coherent oscillation induced by the noise are unmatchable. Thus, a competition between two dynamical modes ap-

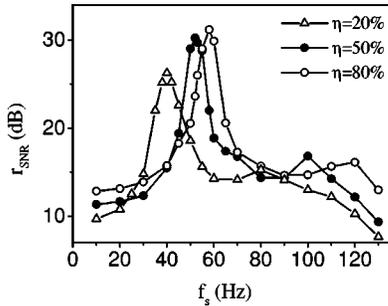


FIG. 10. The SNR of one neuron in the network with $g_{\text{syn}} = 2 \text{ ms/cm}^2$ versus input signal frequency f_s in the case of $D=5$ for $\eta=20\%$, 50% , and 80% , respectively.

pears in the oscillatory dynamics of the system, which results in the coexistence of noise-induced peak and signal-induced peak in the PSD.

Figure 10 summarizes these results, and shows the calculated SNRs from PSDs of one neuron in the network for different frequencies of signals in the case of $D=5$ and $\eta=20\%$, 50% , and 80% , respectively. It is clear that when the frequency of the signal is around that of the spontaneous rhythmic oscillation of the network, a large SNR is obtained. It means that the weak signal with a frequency around that of the rhythmic oscillation can be detected and transduced efficiently by the network. The spontaneous rhythmic oscillation of the network provides itself with an effect of frequency preference to input signal. Physically, such a phenomenon is due to the resonance or cooperation between the noise-induced rhythmic oscillations and the input signals. As in the single neuron case, such a resonance is a coherence phenomenon among the noise, the noise-induced rhythmic oscillation, and the external signal. In addition, it is noted that the frequency sensitivity range of the network is narrower than that in the single neuron case (see Fig. 4). This can be understood from Fig. 8(a). The narrow and high peak in the PSD reflects that due to the coupling, the dynamic motions of the neurons in the network are more coherent than that of the single neuronal case [see Fig. 3(c) and Ref. [22]]. Thus, the frequency selection of the neurons in the network becomes more preferential to some certain external signals.

It is noted that the frequencies corresponding to the maximum of the SNR are about 40 Hz for $\eta=20\%$, 50 Hz for $\eta=50\%$, and 57 Hz for $\eta=80\%$, respectively. That is, for different η , the most frequency sensitive ranges are different. This can be understood clearly from Fig. 8(a) and the above discussion. Obviously, η plays an important role in adjusting the optimal frequency sensitive range. Finally, it is also worth noting that in Fig. 10 there exists a small peak near the twice of the rhythmic oscillation frequency for each curve. This is due to the resonance between the signal and the second-order harmonic of the rhythmic oscillation.

The results of our simulations for the neural network give three conclusions. (1) For a value of $\eta \geq 20\%$, when the coupling strength g_{syn} is large enough, the synchronized rhythmic oscillation can be induced and the coherence of the oscillations can be optimized by the suitable noise. The frequency of the rhythm is roughly in the range of 40–60 Hz.

Besides the noise, η plays an important role in modulating the frequency of the rhythm. (2) The synchronized rhythmic oscillations of the network provide itself with an effect of frequency preference to the input signals. The signals with frequencies around that of the rhythm can be detected sensitively and transmitted effectively. (3) Frequency preference and resonance among the noise, the noise-induced rhythm, and the signal affect largely the ability of the network in processing the external signal.

V. DISCUSSION AND CONCLUSION

In this paper, we make a study on the effect of the external modulations on the ability of the neurons in processing information. The external modulations, which may be caused by various neurotransmitters, the excitatory or the inhibitory synaptic inputs, or the membrane noise, are simplified in general as the sum of I_0 and $I_{\text{noise}}(t)$. This models a fluctuating input around a bias I_0 .

It is well argued that the noise plays an important role in detecting faint, information-bearing signals via SR [3–10]. Here we mainly study the modulatory effect of $I_0 + I_{\text{noise}}(t)$, assumed as an environment, on the dynamics of the HH neurons. Our simulation results demonstrate that such a modulation affects largely the frequency characteristic of the intrinsic oscillations (subthreshold or suprathreshold oscillations), which may be in a range of 30–80 Hz. The intrinsic oscillatory behavior provides the neuron with an effect of frequency selection to external signal. When a weak signal is input to such an oscillator, a cooperative effect among the noise, the noise-induced oscillation, and the signal enhances intensively the ability of the neuron in processing the external signal, especially when the frequency of the signal is around that of the intrinsic oscillation of the neuron. Thus, the neural system may utilize internal or external modulations to adjust itself to an optimally sensitive state for information processing via resonance. Especially, in the low frequency region ($f_s < 25 \text{ Hz}$), we find that the inhibitory modulation (represented by a negative value of I_0) in combination with noise is more beneficial to the neuron for the signal processing than the excitatory modulation (represented by a positive value of I_0). Nevertheless, the excitatory modulation is of great advantage to the neuron in the processing of signals with frequencies in the γ frequency band.

To further examine the effects of the modulation on the frequency sensitivity, we make a study on a neuronal network. The averaged synaptic current of the neurons, $\langle I_{\text{syn}}(t) \rangle$, instead of I_0 , together with the noise current $I_{\text{noise}}(t)$, is a modulation of the network to every individual neuron. When the synaptic coupling strength is large enough, a synchronized rhythmic oscillation or firing of the network are induced and even enhanced by the noise via CR. The frequency of this rhythm is found in a range of 40–60 Hz for $20\% \leq \eta \leq 100\%$. As in the single neuronal case, this synchronized rhythm provides the network with an effect of frequency preference to external signal. When a signal is input to the network, resonance among the noise, the noise-induced rhythmic oscillation and the signal enhances inten-

sively the ability of the network in processing the input signal, especially when the frequency of the signal is near that of the rhythm. The ratio of the excitatory couplings to inhibitory ones of the network can adjust the range of frequency preference of the network.

In conclusion, our results demonstrate the significance of the modulatory effects on the rhythmic activities of neural systems in the process of signal detection and transduction. Resonance and frequency preference associated with the rhythmic oscillations of HH neuronal systems plays an important role in signal processing.

All these results may provide us with an enlightenment as to why the working brain is characterized by various coherent rhythms on characteristic temporal scales. A host of rhythmic oscillators, mutually connected, may tune them-

selves to operate in frequency ranges of some special biological meanings. Resonance and frequency preference may be the basic principles underlying the information processing and even in the realization of different behavioral and perceptual functions of the brain, which needs further study.

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